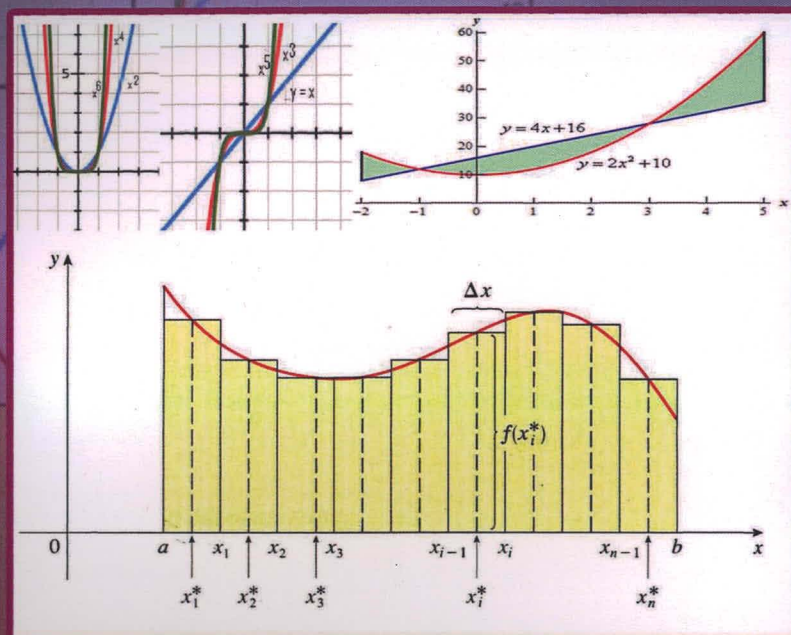


# CALCULUS WITH SINGLE VARIABLE



M. S. H. CHOWDHURY

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EDITED BY  
M. S. H Chowdhury



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